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Reparametrizations and Gauge and General-Coordinate Transformations in String Field Theory

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ABSTRACT

We relate reparametrizations of the parameter σ to point transformations of scalar fields in "loop space," the configuration space of string field theory. Formulas are given for the changes induced by these transformations on the infinite set of "component" spacetime-tensor fields into which a scalar field on loop space may be decomposed. New derivative operators on loop space are defined, motivated by the parametrization-dependence of the mapping from loop space to spacetime. A generalization to loop space of the Einstein-Hilbert Lagrangian is proposed as a candidate for a 2nd-quantized string Lagrangian not tied to any preferred background geometry.

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I. MOTIVATIONS FOR CONSTRUCTING A STRING FIELD THEORY

In describing the quantized dynamics of point particles, we can utilize either a 1st-quantized or a 2nd -quantized formalism ("field theory"). Until recently, work on the dynamics of strings has, with few exceptions [1-3], made exclusive use of 1st quantization. First-quantized methods are well-suited for studying scattering problems, since the propagators of a 1st-quantized theory describe small fluctuations of fields about their background values.

It is in addressing questions involving large fluctuations of the background fields that 2nd quantization is generally of value. The most familiar example of a "large" fluctuation playing an important role in physics is the Higgs mechanism [10]. Another example, of less certain (to date) physical relevance, but of supreme theoretical relevance for string theories, is the spontaneous compactification of spatial dimensions in a theory which is defined with more than three of them to begin with (such as every string theory currently believed to be consistent). In the process of spontaneous compactification it is the gravitational field whose background value is of relevance. Since the gravitational field is one of the "component" dynamical degrees of freedom of a string theory, we must be able to deal with "large fluctuation questions" in string theories if we are to understand how such theories, constrained as they are to twenty-six or ten dimensions, somehow yield our (apparently) four-dimensional world. (Besides gauge-symmetry breaking and spontaneous compatification, there are, of course, other interesting--if perhaps less crucial--questions which 1st-quantized methods seem ill-suited to tackle, e.g.: Do "stringy" black holes have singularities? Do initial singularities occur in "stringy" cosmologies? What is the spectrum of primordial fluctuations in these cosmologies?...)

If we are to construct a string field theory to deal with problems of this nature, it is thus clear that the string field theory we construct should have the following property: The background values of all fields, especially the gravitational field, should be determined dynamically, without any one set of background values holding a preferential position in the very formulation of the theory. Otherwise we may simply be putting in by hand that which should properly be the theory's output.

Unfortunately this property is not possessed by any candidate string field theory with which the present authors are familiar [1-9]. In particular, these theories are constructed in terms of a flat background geometry, and the dynamical gravitational degrees of freedom are deviations from this preferred non-dynamical background. So, issues of fundamental interpretation aside, these theories are technically best suited for studying geometrical questions involving small fluctuations away from flat spacetime; precisely the case where 1st-quantized theory is applicable.

Needless to say, what is required is a 2nd quantized string action not tied to a special background geometry. This action must contain symmetries which, in the particle-field-theory ("zero-slope") limit, reduce to the gauge and general-coordinate invariances which 1st-quantized string theories possess in this limit. One possible source of symmetries is the geometry of loop space. In the following two sections we will study loop space and fields living on loop space, investigating those properties which are independent of the metric of the spacetime from which the loop space is constructed, or of a choice of action for the fields. In the final section we will discuss the relation of loop-space transformations to physical gauge symmetries, and will propose a candidate for a string field theory action formulated independently of a preferred background geometry.

II. REPARAMETRIZATIONS AS TRANSFORMATIONS ON LOOP SPACE

In particle field theory, the dynamical variables are fields which are functions of zero-dimensional subsets of spacetime, i.e., points. In string field theory the fields are functions of one-dimensional subsets of spacetime, i.e., curves $x^\mu(\sigma)$, $\mu=1,\ldots,d$, $0\le\sigma\le\pi$. Strictly speaking, the fields are <u>functionals</u> of $x^\mu(\sigma)$, since it takes a d-fold infinity of numbers to specify the curve $x^\mu(\sigma)$. A functional may be regarded as a limiting case of a function of a finite number of variables:

$$\Psi[x^\mu(\sigma)]\sim\Psi[x^\mu(0.01),x^\mu(0.02),\ldots,x^\mu(\pi)] \tag{2.1}$$

A function of many variables will, in general, have a different value if we change the order of its arguments. For example, say $x^\mu(\sigma)$ describes a closed curve, $x^\mu(\pi)=x^\mu(0)$. Let

$$\sigma\to\bar{\sigma} = \sigma - 0.02 \tag{2.2}$$

and define

$$\bar{x}^\mu(\sigma) \equiv x^\mu(\bar{\sigma}) \tag{2.3a}$$

$$\bar{\Psi}[x^\mu(\sigma)] \equiv \Psi[\bar{x}^\mu(\sigma)] \tag{2.3b}$$

Then

$$\bar{\Psi}[x^\mu(\sigma)]\sim\Psi[x^\mu(\pi-0.01),x^\mu(\pi),\ldots x^\mu(\pi-0.02)] \tag{2.4}$$

and if the initial function Ψ is chosen arbitrarily, it will in general be true that

$$\bar{\Psi}[x^\mu(\sigma)] \neq \Psi[x^\mu(\sigma)]$$

Thus, the space in which the functionals $\Psi[x^\mu(\sigma)]$ live is the space of <u>parametrized</u> <u>curves</u> in d-dimensional spacetime. We will refer to this space as "loop space". (For simplicity we restrict our attention to closed curves, except where indicated otherwise).

An infinitesimal motion of a point in loop space corresponds to an infinitesimal displacement in spacetime of each point of the curve $x^\mu(\sigma)$ to yield the curve $\bar{x}^\mu(\sigma)$;

$$\bar{x}^\mu(\sigma) = x^\mu(\sigma) + \epsilon v^\mu(\sigma) \tag{2.5}$$

where ϵ is an infinitesimal parameter and $v^\mu(\sigma)$ is a spacetime vector at the point on the curve x^μ with parameter label σ. So, a vector in loop space at the loop-space point $x^\mu(\sigma)$ is a vector <u>field</u> in spacetime defined along the parametrized curve in spacetime $x^\mu(\sigma)$. To each parametrized curve in spacetime we can associate a different spacetime vector field, thus obtaining a loop space vector field $v^\mu[x](\sigma)$.

We can now identify certain point tranformations in loop space, $x^\mu(\sigma)\to\bar{x}^\mu(\sigma)$, with "reparametrizations" of σ, $\sigma\to\bar{\sigma}$. A point transformation in loop space is a reparametrization if it maps each point $x^\mu(\sigma)$ on the original curve to a point $\bar{x}^\mu(\sigma)$ which is also a point of the original curve $x^\mu(\sigma)$. I.e., $x^\mu(\sigma)\to\bar{x}^\mu(\sigma)$ is a reparametrization if there exists a function $\bar{\sigma}(\sigma)$ such that, for all σ,

$$\bar{x}^\mu(\sigma) = x^\mu(\bar{\sigma}(\sigma)) \; . \tag{2.6}$$

Since we can define reparametrizations which differ from one curve to another, we should really write $\bar{\sigma}[x](\sigma)$; or, for infinitesimal reparametrizations,

$$\bar{\sigma} = \sigma + \epsilon\, g[x](\sigma) \; . \tag{2.7}$$

Consider an infinitesimal point transformation of the form (2.5); if this transformation is an infinitesimal reparametrization, then, using (2.7),

$$\begin{aligned} x^\mu(\sigma) + \epsilon v^\mu[x](\sigma) &= x^\mu(\sigma+\epsilon g[x](\sigma)) \\ &\approx x^\mu(\sigma)+\epsilon g[x](\sigma)\,\frac{dx^\mu}{d\sigma} \; . \end{aligned} \tag{2.8}$$

So, if a vector field in loop space is to correspond to an infinitesimal reparametrization, it must be of the form

$$v^\mu_R[x](\sigma) = g[x](\sigma)x'^\mu(\sigma) \tag{2.9}$$

where

$$x'^{\mu}(\sigma) \equiv \frac{dx^{\mu}}{d\sigma} \quad . \tag{2.10}$$

That is, the corresponding spacetime vector field $v^{\mu}[x](\sigma)$ must be everywhere tangent to the curve $x^{\mu}(\sigma)$, as we might intuitively expect.

Having identified point transformations in loop space corresponding to infinitesimal σ reparametrizations, we are now in a position to define the action of reparametrizations on scalar functions on loop space. If $\Psi[x]$ is a scalar functional of the curve $x^{\mu}(\sigma)$, then the "reparametrized" functional $\bar{\Psi}[x]$ is a scalar functional which has the same value at $x^{\mu}(\sigma)$as $\Psi[x]$ has at $\bar{x}^{\mu}(\sigma)=x^{\mu}(\sigma)+\epsilon v_R^{\mu}[x](\sigma)$:

$$\bar{\Psi}[x^{\mu}] \equiv \Psi[\bar{x}^{\mu}] = \Psi[x^{\mu}+\epsilon v_R^{\mu}[x]] \tag{2.11}$$

To $O(\epsilon)$,

$$\Psi[\bar{x}\ ]^{\mu}= \Psi[x] + \epsilon\ \Delta_{v_R}\Psi[x] \tag{2.12}$$

where the derivative operator Δ_{v_R} [11],

$$\Delta_{v_R}\Psi[x] \equiv \int_0^{\pi} d\sigma v_R^{\mu}[x](\sigma)\ \frac{\delta\Psi[x]}{\delta x^{\mu}(\sigma)} \tag{2.13}$$

can of course be defined for any loop-space vector field $v^{\mu}[x](\sigma)$, not just those of the form (2.9).

III. REPARAMETRIZATIONS AS TRANSFORMATIONS OF TENSOR FIELDS ON SPACETIME

To investigate the relation between infinitesimal reparametrizations and linearized gauge transformations we must first describe the relation between $\Psi[x(\sigma)]$ and its "component" tensor fields [1,2]. Expand $\Psi[x(\sigma)]$ in a Taylor series about a point in loop space which also happens to be a point in spacetime; i.e., the zero-length curve

$$\overset{\circ}{x}{}^{\mu} \equiv \int_0^{\pi} d\sigma x^{\mu}(\sigma) \quad . \tag{3.1}$$

Then

$$\Psi[x^{\mu}] = \Psi[\overset{\circ}{x}{}^{\mu}] + \int_0^{\pi} d\sigma \left.\frac{\delta\Psi[x]}{\delta x^{\mu}(\sigma)}\right|_{x^{\mu}=\overset{\circ}{x}{}^{\mu}} (x^{\mu}(\sigma)-\overset{\circ}{x}{}^{\mu})$$
$$+ \frac{1}{2!}\int_0^{\pi} d\sigma_1 d\sigma_2 \left.\frac{\delta^2\Psi[x]}{\delta x^{\mu_1}(\sigma_1)\delta x^{\mu_2}(\sigma_2)}\right|_{x^{\mu}=\overset{\circ}{x}{}^{\mu}} (x^{\mu_1}(\sigma_1)-\overset{\circ}{x}{}^{\mu_1})(x^{\mu_2}(\sigma_2)-\overset{\circ}{x}{}^{\mu_2}) + \ldots \tag{3.2}$$

Choose as a basis for functions of σ, $0 \le \sigma \le \pi$, a set of functions satisfying

$$\int_0^{\pi} d\sigma f^{\dagger}_{\ell}(\sigma) f_m(\sigma) = \delta_{\ell m} \tag{3.3a}$$

$$\sum_{\ell} f_{\ell}(\sigma_1) f_{\ell}(\sigma_2) = \delta(\sigma_1 - \sigma_2) \tag{3.3b}$$

$$f_0(\sigma) = f^{\dagger}_0(\sigma) = \frac{1}{\sqrt{\pi}} \quad . \tag{3.3c}$$

Then $\Psi\,[x^{\mu}(\sigma)]$ may be expressed in terms of an infinite number of tensor fields which live on the spacetime manifold with coordinates $\overset{\circ}{x}{}^{\mu}$:

$$\Psi[x^{\mu}(\sigma)] = \sum_{I=0}^{\infty} \sum_{\ell_1,\ldots\ell_I \neq 0} x^{\mu_1}_{\ell_1} \ldots x^{\mu_I}_{\ell_I} B^{\ell_1\ldots\ell_I}_{\mu_1\ldots\mu_I}(\overset{\circ}{x}) \tag{3.4}$$

where (for $\ell \neq 0$)

$$B^{\ell_1\cdots\ell_I}_{\mu_1\cdots\mu_I}(\mathring{x})$$

$$= \frac{1}{I!}\int_0^\pi d\sigma_1\cdots d\sigma_I\left\{\frac{\delta}{\delta x^{\mu_1}(\sigma_2)}\cdots\frac{\delta}{\delta x^{\mu_I}(\sigma_I)}\Psi[x]\right\}\Bigg|_{x=\mathring{x}} f_{\ell_1}(\sigma_1)\cdots f_{\ell_1}(\sigma_I) \qquad (3.5)$$

and

$$x^\mu_\ell \equiv \int_0^\pi d\sigma f^\dagger_\ell(\sigma)x^\mu(\sigma) \quad . \qquad (3.6)$$

We now study how the components

$$B^{\ell_1\cdots\ell_I}_{\mu_1\cdots\mu_I}$$

of Ψ change under an infinitesimal reparametrization of the form (2.8). We will restrict our attention to curve-independent reparametrizations; that is, reparametrizations generated by loop-space vector fields of the form

$$v^\mu_r(\sigma) = g(\sigma)x'^\mu(\sigma) \qquad (3.7)$$

rather than the more general form (2.8) (Operators of this form appear as parts of the Virasoro operators of first-quantized string theory, as we discuss in section IV. Reparametrization fields of the more general form (2.8) may be of importance, but we shall not consider them here).

Define

$$g_\ell \equiv \int_0^\pi d\sigma f^\dagger_\ell(\sigma)g(\sigma) \qquad (3.8a)$$

so

$$g(\sigma) = \sum_\ell g_\ell f_\ell(\sigma) \quad . \qquad (3.8b)$$

Using the above definition, (2.13), (3.4),(3.7), and the relations

$$\frac{\delta x^\mu_\ell}{\delta x^\nu(\sigma)} = \delta^\mu_\nu f^\dagger_\ell(\sigma) , \qquad \ell\neq 0 \tag{3.9a}$$

$$\frac{\delta x^\mu}{\delta x^\nu(\sigma)} = \delta^\mu_\nu \frac{1}{\pi} \tag{3.9b}$$

which follow from (3.1) and (3.6), we find that

$$\Delta_{V_R}\Psi[x] = \sum_{I=1} \sum_{\ell_1 \ldots \ell_I \neq 0} x^{\mu_1}_{\ell_1} \ldots x^{\mu_I}_{\ell_I} \tag{3.10}$$

$$\cdot\left\{\frac{2i\ell_1}{\pi}\, g_{-\ell_1} \frac{\partial}{\partial \mathring{x}^{\mu_1}} B^{\ell_2\ldots\ell_I}_{\mu_2\ldots\mu_I}(\mathring{x}) + \sum_b g_b \frac{2iI\ell_1}{\sqrt{\pi}}\right.$$

$$\left.\cdot(1-\delta_{\ell_1,-b}) B^{\ell_1+b_1\ldots\ell_I}_{\mu_1\ldots\mu_I}(\mathring{x})\right\}$$

In (3.10) we have chosen

$$f_\ell(\sigma) = \frac{1}{\sqrt{\pi}}\, e^{2i\ell\sigma} \tag{3.11}$$

and used the relation

$$B^{\ell_1\ldots\ell_a\ldots\ell_b\ldots\ell_I}_{\mu_1\ldots\mu_a\ldots\mu_b\ldots\mu_I} = B^{\ell_1\ldots\ell_b\ldots\ell_a\ldots\ell_I}_{\mu_1\ldots\mu_b\ldots\mu_a\ldots\mu_I}$$

which follows from the definition (3.5).

The components of any scalar function on loop space may be obtained using (3.5) or, equivalently, by taking derivatives with respect to the x^μ_ℓ's and evaluating the result at $x^\mu_\ell = 0$ (i.e., at $x^\mu(\sigma) = \mathring{x}^\mu$). The first three components of (2.12) are, therefore

$$(\bar{\Psi} - \Psi - \varepsilon\, \Delta_{v_r} \Psi)\Big|_{x^\mu_\ell = 0} = 0 \tag{3.12}$$

$$\frac{\partial}{\partial x^\alpha_m}\,(\bar{\Psi} - \Psi - \varepsilon\, \Delta_{v_r} \Psi)\Big|_{x^\mu_\ell = 0} = 0 \tag{3.13}$$

$$\frac{\partial}{\partial x^{\alpha_1}_{m_1}}\,\frac{\partial}{\partial x^{\alpha_2}_{m_2}}\,(\bar{\Psi} - \Psi - \varepsilon\, \Delta_{v_r} \Psi)\Big|_{x^\mu_\ell = 0} = 0 \tag{3.14}$$

Using (3.4) and (3.10) and defining

$$\delta B^{m_1 \ldots m_I}_{\alpha_1 \ldots \alpha_I}(\mathring{x}) = \bar{B}^{m_1 \ldots m_I}_{\alpha_1 \ldots \alpha_I}(\mathring{x}) - B^{m_1 \ldots m_I}_{\alpha_1 \ldots \alpha_I}(\mathring{x}) \tag{3.15}$$

where $\bar{B}^{m_1 \ldots m_I}_{\alpha_1 \ldots \alpha_I}$ $(B^{m_1, \ldots m_I}_{\alpha_1 \ldots \alpha_I})$ is a component of $\bar{\Psi}(\Psi)$, (3.12)-(3.14) become

$$\delta B(\mathring{x}) = 0 \tag{3.16}$$

$$\delta B^\alpha_m(\mathring{x}) = \frac{2im\varepsilon}{\sqrt{\pi}}\left\{\frac{g_{-m}}{\sqrt{\pi}}\,\frac{\partial B(\mathring{x})}{\partial \mathring{x}^\alpha} + \sum_b g_b (1-\delta_{m,-b}) B^{m+b}_\alpha(\mathring{x})\right\} \tag{3.17}$$

$$\delta B^{\alpha_1 \alpha_2}_{m_1 m_2}(\mathring{x}) = \frac{4i\varepsilon}{\sqrt{\pi}}\,(1-\delta_{\mu_{m_1},0})(1-\delta_{\mu_{m_2},0})$$

$$\cdot\Big\{m_1\Big[\frac{g_{-m_1}}{2\sqrt{\pi}}\,\frac{\partial}{\partial \mathring{x}^{\alpha_1}}\,B^{m_2}_{\alpha_2}(\mathring{x}) + \sum_b g_b (1-\delta_{m_1,-b}) B^{m_1+b,\, m_2}_{\alpha_1 \quad \alpha_2}(\mathring{x})\Big]$$

$$+ m_2\Big[\frac{g_{-m_2}}{2\sqrt{\pi}}\,\frac{\partial}{\partial \mathring{x}^{\alpha_2}}\,B^{m_1}_{\alpha_1}(\mathring{x}) + \sum_b g_b (1-\delta_{m_2,-b}) B^{m_2+b,\, m_1}_{\alpha_2 \quad \alpha_1}(\mathring{x})\Big]\Big\} \tag{3.18}$$

In (3.16)-(3.18), we have computed the changes in the components

$$B^{\ell_1\cdots\ell_I}_{\mu_1\cdots\mu_I}(\overset{\circ}{x})$$

of Ψ with regard to a fixed basis

$$x^{\mu_1}_{\ell_1}\cdots x^{\mu_I}_{\ell_I}\ ;$$

schematically,

$$\Delta\ \Psi \sim \delta B^{\ell}_{\mu}\cdot x^{\mu}_{\ell}\quad . \tag{3.19}$$

There are other ways to define the effect of a derivative operator on the component fields. In general, the x^{μ}_{ℓ}'s are changed under reparametrization. If

$$\bar{\sigma} = \sigma + \varepsilon g(\sigma) \tag{3.20}$$

and

$$\bar{x}^{\mu}(\sigma) = x^{\mu}(\sigma) = x^{\mu}(\sigma)\rightarrow \varepsilon g(\sigma)x'^{\mu}(\sigma) \tag{3.21}$$

then

$$\bar{x}^{\mu}_{\ell} = x^{\mu}_{\ell} + \varepsilon\int_0^{\pi}d\sigma f^{\dagger}_{\ell}(\sigma)g(\sigma)x'^{\mu}(\sigma) \tag{3.22}$$

(In obtaining (3.22) we require that the functions $f_{\ell}(\sigma)$ retain their form under reparametrization, i.e.,

$$\bar{f}_{\ell}(\bar{\sigma}) = f_{\ell}(\bar{\sigma}) \tag{3.23}$$

To do otherwise would be to introduce at the outset a distinction between different choices of parametrization). Even the spacetime coordinates $\overset{\circ}{x}^{\mu}$ are not invariant under arbitrary reparametrizations:

$$\bar{\mathring{x}}^{\mu} = \mathring{x}^{\mu} + \frac{\varepsilon}{\pi} \int_0^{\pi} d\sigma g(\sigma) x'^{\mu}(\sigma) \quad . \tag{3.24}$$

For any reparametrization (3.20), (3.21), define the derivative operator $\mathscr{D}_g$ by the relations

$$\mathscr{D}_g \, x^{\mu}_{\ell} = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} (\bar{x}^{\mu}_{\ell} - x^{\mu}_{\ell}) \ ; \ \ell \neq 0 \tag{3.25a}$$

$$\mathscr{D}_g \, \mathring{x}^{\mu}_{\ell} = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} (\bar{\mathring{x}}^{\mu}_{\ell} - \mathring{x}^{\mu}_{\ell}) \ . \tag{3.25a}$$

Using (3.23)-(3.25) we find that

$$\mathscr{D}_g x^{\mu}_{\ell} = \sum_{m} x^{\mu}_{m} [\frac{2im}{\sqrt{\pi}} g_{\ell-m}] \ , \quad \ell \neq 0 \tag{3.26}$$

$$\mathscr{D}_g \mathring{x}^{\mu}_{\ell} = \sum_{m} x^{\mu}_{m} [\frac{2im}{\pi} g_{-m}] \quad . \tag{3.27}$$

To obtain the action of $\mathscr{D}_g$ on the components

$$B^{\ell_1 \cdots \ell_I}_{\mu_1 \cdots \mu_I}(\mathring{x}) \ ,$$

we require that $\mathscr{D}_g$ reduce to the directional derivative in the direction $v^{\mu}_{r}[x](\sigma) = g(\sigma)x'^{\mu}(\sigma)$ when acting on scalars,

$$\mathscr{D}_g \Psi = \Delta_{v_r} \Psi \ , \tag{3.28}$$

and that $\mathscr{D}_g$ obey the Leibniz rule; schematically,

$$\mathscr{D}_g \, \Psi = (\mathscr{D}_g B^{\ell}_{\mu}) \cdot x^{\mu}_{\ell} + B^{\ell}_{\mu} \cdot (\mathscr{D}_g x^{\mu}_{\ell}) \quad . \tag{3.29}$$

We find that

$$B^{\ell_1\cdots\ell_I}_{\mu_1\cdots\mu_I}(\overset{\circ}{x}) = (1-\delta_{I,0})\sum_{k=1}^{I} \frac{2i\ell_k}{\pi I} g_{-\ell_k} \frac{\partial}{\partial x^{\mu_k}} B^{\ell_1\cdots\ell_{k-1}\,\ell_{k+1}\cdots\ell_I}_{\mu_1\cdots\mu_{k-1}\,\mu_{k+1}\ldots\mu_I}(\overset{\circ}{x}) , \tag{3.30}$$

for example:

$$B(\overset{\circ}{x}) = 0 \tag{3.31}$$

$$\mathscr{D}_g B^{\ell}_{\mu}(\overset{\circ}{x}) = \frac{2i\ell}{\pi} g_{-\ell} \frac{\partial B(\overset{\circ}{x})}{\partial \overset{\circ}{x}^{\mu}} \tag{3.32}$$

$$\mathscr{D}_g B^{\ell_1\,\ell_2}_{\mu_1\,\mu_2}(\overset{\circ}{x}) = \frac{i\ell_1}{\pi} g_{-\ell_1} \frac{\partial}{\partial \overset{\circ}{x}^{\mu_1}} B^{\ell_2}_{\mu_2}(x) + \frac{i\ell_2}{\pi} g_{-\ell_2} \frac{\partial}{\partial \overset{\circ}{x}^{\mu_2}} B^{\ell_1}_{\mu_1}(\overset{\circ}{x}) \tag{3.33}$$

Equation (3.33) may be reexpressed in terms of its antisymmetric and symmetric parts:

$$\mathscr{D}_g B^{\ell_1\ell_2}_{[\mu_1\mu_2]}(\overset{\circ}{x}) = \partial_{[\mu_1}\,{}^{(-)}\xi^{\ell_1\ell_2}_{\mu_2]}(\overset{\circ}{x}) \tag{3.34}$$

and

$$\mathscr{D}_g B^{\ell_1\ell_2}_{(\mu_1\mu_2)}(\overset{\circ}{x}) = \partial_{(\mu_1}\,{}^{(+)}\xi^{\ell_1\ell_2}_{\mu_2)}(\overset{\circ}{x}) \tag{3.35}$$

where

$${}^{(\pm)}\xi^{\ell_1\ell_2}_{\mu} \equiv \frac{i\ell_1}{\pi} g_{-\ell_1} B^{\ell_1}_{\mu} \pm \frac{i\ell_2}{\pi} g_{-\ell_2} B^{\ell_1}_{\mu} . \tag{3.36}$$

The transformations (3.32), (3.34) and (3.35) are of the forms of linearized gauge transformations of Maxwell, Kalb-Ramond and gravitational fields, respectively.

(In a physical closed-string field the vector component is removed upon imposing--either by restricting the space of states or by a projection operator in the Lagrangian--the requirement that Ψ be invariant under <u>uniform</u> curve-independent reparametrizations; i.e.,

$$\Delta_{v_0}\Psi = 0 \tag{3.37}$$

where

$$v_0^\mu[x](\sigma) = g_0 x'^\mu(\sigma), \quad g_0 = \text{constant} \quad . \tag{3.38}$$

The reader can verify that (3.37) implies

$$B^{\ell_1 \cdots \ell_I}_{\mu_1 \cdots \mu_I}(\mathring{x}) = 0 \quad \text{unless } \ell_1 + \ldots + \ell_I = 0 \tag{3.39}$$

so, in particular, $B^{\ell}_{\mu}(\mathring{x}) = 0$ since $\ell \neq 0$.

For open strings the restriction (3.39) does not arise, since, for these, $g(\sigma) = g_0$ = constant is not a reparametrization, except for $g_0 = 0$. For open strings

$$\tilde{x}^\mu(0) = x^\mu(0) \quad , \quad \tilde{x}^\mu(\pi) = x^\mu(\pi) \tag{3.40}$$

so (2.8),(2.9), and (3.40) imply that

$$g(0) = g(\pi) = 0 \quad . \tag{3.41}$$

We cannot invoke here the 1st-quantized equations of motion, $x'^\mu(0)=x'^\mu(\pi)=0$, since we are dealing with the field theory and $x^\mu(\sigma)$ is, not a dynamical variable, but an element of the set which indexes the dynamical variables Ψ. So, we see that the only uniform reparametrization of open curves consistent with (3.40) is the identity, $g(\sigma) = 0$).

What is the infinitesimal transformation of Ψ such that the <u>components</u> of this transformation (i.e., with respect to fixed x^μ_ℓ's) are of the form (3.30)? That is, what operator $\mathbf{D}_g$ satisfies

$$\varepsilon\, \mathbf{D}_g \Psi = (\varepsilon \mathscr{D}_g B^{\ell}_{\mu}) \cdot x^\mu_\ell \;? \tag{3.42}$$

The required operator has the form

$$\mathbf{D}_g = \frac{2i}{\pi} \left(\sum_m m\, x^\mu_m g_{-m}\right) \frac{\partial}{\partial \overset{\circ}{x}{}^\mu} \quad . \tag{3.43}$$

It can be easily checked that, unless $g(\sigma)$ is identically zero, $\mathbf{D}_g$ is not a curve-independent reparametrization. That is, there is no function $q(\sigma)$ such that

$$\mathbf{D}_g = \Delta_{qx'} = \int_0^\pi d\sigma\; q(\sigma) x'(\sigma) \frac{\delta}{\delta x^m(\sigma)} \quad . \tag{3.44}$$

However, $\mathbf{D}_g$ does have a simple interpretation. Using (3.27b) and (3.43), we see that

$$\varepsilon\, \mathbf{D}_g = \varepsilon\, \mathscr{D}_g\, \overset{\circ}{x}{}^\mu \frac{\partial}{\partial \overset{\circ}{x}{}^\mu} = (\bar{\overset{\circ}{x}}{}^\mu - \overset{\circ}{x}{}^\mu) \frac{\partial}{\partial \overset{\circ}{x}{}^\mu} \quad . \tag{3.45}$$

So, $\varepsilon\, \mathbf{D}_g$ is the spacetime translation operator which translates by an amount proportional to the shift in spacetime coordinate $\overset{\circ}{x}{}^\mu$ associated with a reparametrization $\sigma \to \sigma + \varepsilon g(\sigma)$.

IV. DISCUSSION

Is either the Δ or $\mathbf{D}$ transformation a viable candidate to yield linearized gauge transformations in a physical string field theory? That is, should one attempt to construct loop space actions, functionals of $\Psi[x^\mu(\sigma)]$, which, at least linearly, are invariant under

$$\Psi \to \Psi + \varepsilon \Delta_{v_r} \Psi \tag{4.1}$$

or

$$\Psi \to \Psi + \varepsilon \mathbf{D}_g \Psi \ ? \tag{4.2}$$

Invariance under (4.1) or (4.2) corresponds to invariance under component-field transformations such as (3.16)-(3.18) or (3.31)-(3.35). Although, as has been noted, (3.32),(3.34), and (3.35) are similar in appearance to familiar gauge-transformation laws, both (4.1) and (4.2) differ from the usual gauge transformations in an important respect; the transformation parameters on the right-hand sides are themselves proportional to the fields which are being transformed, rather than totally arbitrary functions. Theories possessing invariance under (4.1) or (4.2) would thus be very different from conventional gauge theories [12]. If we want the usual electromagnetic, gravitational, etc., linearized gauge invariances to be contained in our theory, we must replace (4.1) and (4.2) with

$$\Psi \to \Psi + \varepsilon \Delta_{v_r} \Omega \tag{4.3}$$

$$\Psi \to \Psi + \varepsilon \mathbf{D}_g \Omega \tag{4.4}$$

where Ω is an arbitary scalar function on loop space. (Admittedly, in doing so we lose the purely geometric interpretation which (4.1), (4.2) have, although the operators Δ_{v_r} and $\mathbf{D}_g$ are still associated with geometric transformations in loop space). The equations for the components of tranformations (4.3) and (4.4) are identical to those for the components of (4.1) and (4.2), except that, on the right-hand sides,

$$B^{\ell_1 \cdots \ell_I}_{\mu_1 \cdots \mu_I}(\mathring{x})$$

is replaced by

$$\omega^{\ell_1 \cdots \ell_I}_{\mu_1 \cdots \mu_I}(\mathring{x}) \quad ,$$

where

$$\Omega[x] = \sum_{I=0}^{\infty} \sum_{\ell,\ldots \ell_I \neq 0} x^{\mu_1}_{\ell_1} \ldots x^{\mu_I}_{\ell_I}\, \omega^{\ell_1 \cdots \ell_I}_{\mu_1 \cdots \mu_I}(\mathring{x}) \quad . \tag{4.5}$$

For example, (3.32) becomes

$$
{}_g B^{\lambda}_{\mu}(\mathring{x}) = \frac{2i\lambda}{\pi}\, g_{-\lambda}\, \frac{\partial\omega(\mathring{x})}{\partial\mathring{x}^{\mu}} \quad . \tag{4.6}
$$

Even with this improvement, the rule $\Psi\to\Psi+\varepsilon \mathbf{D}_g\Omega$ is unsuitable in another way for use as a gauge-transformation law. Under this rule, all the component tensor fields transform as gauge fields. So, a theory with this symmetry would have an infinite number of physical gauge fields.

As for $\Psi\to\Psi+\varepsilon\Delta_{v_r}\Omega$, comparison with first-quantized string theory suggests that this is only part of the correct rule. In the first-quantized theory in flat spacetime a crucial role is played by the Virasoro operators [13]. Each of these operators is equal to the sum of a Δ_{v_r} operator, for a suitable $g(\sigma)$, with another piece which is a Fourier transform with respect to σ of the operator

$$
-\eta^{\mu\nu}\frac{\delta}{\delta x^{\mu}(\sigma)}\,\frac{\delta}{\delta x^{\nu}(\sigma)} + \frac{1}{4\pi^2\alpha'^2}\,\eta_{\mu\nu}\,x'^{\mu}(\sigma)x'^{\nu}(\sigma) \tag{4.7}
$$

($\eta_{\mu\nu}$ is the Minkowski metric, and α' is a constant). In the context of string field theory it is not clear how to give these operators a geometrical interpretation [14]. There is of, course, a formal similarity with the mass-shell operator for point particles,

$$
-\eta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\,\frac{\partial}{\partial x^{\nu}} + m^2 \tag{4.8}
$$

in that both (4.7) and (4.8) arise from the constraint equations satisfied by the respective canonical momenta,

$$
P^{\mu}(\sigma)P_{\mu}(\sigma) + \frac{1}{4\pi^2\alpha'^2}\,\eta_{\mu\nu}\,x'^{\mu}(\sigma)x'^{\nu}(\sigma) = 0 \quad , \tag{4.9}
$$

$$
p^{\mu}p_{\mu} + m^2 = 0 \quad ,
$$

as consequences of the choice of a parametrization-invariant action for the respective classical objects [13],

$$S(\text{string}) \sim \int d\sigma\, d\tau[-(\dot{x})^2(x')^2 + (\dot{x}\cdot x')^2]^{1/2} \,, \tag{4.11}$$

$$S(\text{particle}) \sim \int d\tau\, [-\dot{x}^2]^{1/2} \,. \tag{4.12}$$

This analogy suggests that the Nambu action (4.11) be regarded as a distance function in loop space, and that any metric tensor on "curved" loop space should have the property that the "interval" between the loop-space points $x^\mu(\sigma)$, $y^\mu(\sigma)$ is equal, at least for "nearby" points, to the area of the minimal surface between them.

Having seen that loop-space geometry may bear at least some relation to physical gauge symmetries, we are motivated to propose a purely geometric candidate for a background-geometry-independent formulation of the string field theory action. Our proposal is a modification of an action invented by Freund and Nepomechie [15] in studying the geometry of Kalb-Ramond fields, and its potential applicability to string theory was originally noted by them. These authors take as their action the Einstein-Hilbert action constructed from a metric tensor living on a space even larger than loop space, namely, the fiber bundle (loop space) $\otimes$ U(1). After certain ad hoc restrictions are imposed on the form of the metric and the $x^\mu_\ell \to 0$ limit is taken, this action reduces to the bosonic part of the action for ten-dimensional supergravity.

Since Ψ contains objects,

$$B^{\ell m}_{(\mu\nu)} \,,$$

with transformation properties similar to the linearized gauge transformation properties of a metric perturbation $h_{\mu\nu}$, it seems unwarranted to introduce an additional field in the theory to describe the geometry. Indeed, one of the beauties of string theory is precisely the fact that gravitational degrees of freedom arise from the string degrees of freedom. We therefore suggest that one take as the loop-space metric tensor, out of which to construct the loop-space Einstein-Hilbert action, the second variational derivative of the scalar string field itself (not restricted to $x^\mu_\ell = 0$!):

$$g_{\mu_1\mu_2}(\sigma_1,\sigma_2) = \frac{\delta^2\Psi[x]}{\delta x^{\mu_1}(\sigma_1)\delta x^{\mu_2}(\sigma_2)} \quad . \tag{4.13}$$

Ψ is, in general, complex, so it may not be necessary to augment loop space with an additional U(1) fiber. The results of [15] seem to indicate that such a theory will have a suitable particle-field-theory ($x^\mu_\lambda \to 0$) limit; its properties as a string field theory ($x^\mu_\lambda \neq 0$) are currently under investigation.

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